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# Concentration polarization and permeate flux variation in a vibration enhanced reverse osmosis membrane module

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## ABSTRACT

The performance of a vibration enhanced reverse osmosis membrane module for desalination of artificial seawater was investigated with computational fluid dynamics (CFD) simulations and experiments. The computational model couples fluid dynamics with solutes transport inside the full length domain containing 'zigzag' spacers using a two dimensional, transient Large Eddy Simulation turbulence model. Both the local concentration dependent solute properties and variation of permeate flux over the membrane surface were predicted with the model. Membrane local permeate flux, concentration polarization, shear rate, and mass transfer were also calculated. The results suggest concentration polarization in the seawater desalination process could be reduced by imposing vibration on the reverse osmosis membrane. It was determined that the higher the vibration frequency the higher the membrane permeate flux while keeping the vibration amplitude constant. The CFD simulation predictions are validated against experimental data of permeate fluxes with good agreement.

## 1. Introduction

High pressure seawater reverse osmosis (RO) is one of the most economic approaches for desalination. One challenge of RO membrane system is the concentration polarization phenomenon caused by the accumulation of rejected solutes and particles on the membrane surface. Concentration polarization will reduce permeate flux through the membrane and accelerate the membrane fouling, which limits the membrane lifetime [1]. Therefore, accurate predictions of the solute local concentration profile, and concentration polarization improvement are highly desired for improved design and operation of membrane desalination systems.

As mentioned in the review by Ghidossi et al. [2], computational fluid dynamics (CFD) is becoming more important in RO membrane science since it can provide more detailed flow and solute distribution information, especially when combining CFD simulation with laboratory membrane module experiments.

Experiments with ladder-type, spacer-filled rectangular channels with a tracer injection flow visualization technique were performed by Geraldes et al. [3] to study the effects of spacer parameters on membrane performance. The flow visualization and friction factor measurements showed that critical Reynolds numbers increase with the decrease of the distance between spacers.

Fimbres-Weihs et al. [4] studied transient unsteady flows inside two

dimensional spacer-filled channels using the CFD method. Unsteady flow patterns were observed inside the feed channels at Reynolds number of 841 and 1683. Vortices formed and grew behind the feed spacer moving downstream with the bulk flow. The regions where the fluid flowed near the membrane wall contained both high mass transfer rate and high shear rate.

Concentration polarization profiles for three single salt solutions of NaCl,  $CaCl_2$  and  $Na_2SO_4$  in a rectangular RO and laboratory plate-and-frame RO membrane channel were numerically investigated by Lyster and Cohen [5]. They proved the importance of coupling fluid dynamics and mass transfer governing equations and accounting for permeate flux variability along the membrane surface for studying membrane concentration polarization profiles.

High shear rate at the membrane surface to reduce solute/particle deposition is known to be an effective way to reduce concentration polarization and increase the permeate flux in cross-flow membrane filtration [6–9]. Imposing motion of the membrane is one way to increase the shear rate at the membrane surface. Yeh and Cheng [10] used boundary layer theory to analyze membrane surface slip effect on the permeate flux in ultra-filtration. They found that the mass-transfer rate as well as permeate flux increase with the increase of the membrane slip velocity, which reduces the concentration polarization. A similar conclusion was also found by Chellam et al. [11] according to their 2D simulation of a channel bounded by one porous membrane

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wall boundary subject to uniform suction.

The vibratory shear enhanced filtration process (VSEP) [12] is a commercial unit that relies on rapid membrane motion to induce large shear rate. The VSEP system can be treated as a disk-shaped desalination cell attached to a central shaft, utilizing high frequency torsional vibration of membrane to impose membrane motion. AI Akoum et al. [13] performed experiments to investigate the performance of VSEP module in yeast microfiltration (MF) and bovine albumin ultrafiltration (UF). They found that for both yeast MF and bovine albumin UF, the higher the membrane shear rate, the higher the permeate flux with torsional vibration frequency in the range of 56 Hz to 61 Hz. Experimental research investigated by Shi and Benjamin [14] indicated that inorganic fouling can be reduced by high shear rate at the membrane surface imposed by the torsional vibration of VSEP system. A study proposed by Shi and Benjamin [15] revealed that an increase in vibration amplitude will decrease the membrane fouling and increase the rejection of most solutes. Varying both vibration frequency and amplitude showed that membrane fouling rate was almost invariant as long as the averaged shear rate is the same. However, the vibration amplitude of points near the center of shaking will have less effective shaking amplitude for the torsional vibration utilized in the VSEP system. This may not fully utilize the vibration benefit on salt concentration polarization reduction and fouling mitigation.

In this study, a novel vibration enhanced desalination technique was proposed to address the RO membrane concentration polarization problem, with the aim of increasing membrane permeate flux and reducing membrane salt concentration. In this technique, a RO membrane desalination cell is driven by linear actuators to impose axial rapid membrane vibration and large membrane surface shear rates. Fifteen cases for different vibration and flow conditions were first performed in the vibration enhanced reverse osmosis membrane desalination setup to experimentally study the effect of vibration on RO membranes for seawater desalination. Detailed hydrodynamics were then analyzed using transient CFD simulation consisting of the fully coupled fluid dynamics and solute transportation governing equations performed for the 2D spacer-filled, full length vibration membrane channel. Finally, the simulation results were validated by the corresponding experimental observations.

#### 2. Experimental work

#### 2.1. Simulated seawater feed solution composition and membranes

Distilled water was used to prepare the artificial seawater feed solution. The composition of feed solution is listed in Table 1. All chemicals used in feed water preparation were reagent grade and purchased from Fisher Scientific.

The membrane used in all tests had 20 cm length  $\times$  3 cm width, yielding an effective membrane area of 60 cm<sup>2</sup>. Specifications for the membrane are summarized in Table 2 [17].

#### 2.2. Vibratory RO membrane test cell set up

Fig. 1 shows the vibratory RO membrane cell module, which is employed for all experiments. This membrane cell module has two separate parts. The upper part contains the high pressure feed channel, a

Table 1Feed water composition.

Chemical	mol/L
NaCl MgSO4 CaCl2 Na-SO4	0.4187 0.0503 0.0342 0.0172

Table 2Specification of the membrane.

Membrane	SWC6 MAX
Vendor	Hydranautics
Material	Composite polyamide
Permeate flow <sup>a</sup>	7.2 lpd
Salt rejection	99.8%
pH range, continuous	2–11
Max Cl <sub>2</sub> concentration	0.1 ppm
Max temperature	318 K

<sup>a</sup> At 55 bar applied pressure.

feed water inlet port and a retentate outlet port. The height of the feed channel is 0.78 mm and the feed spacer is placed inside the feed channel. The feed spacer used in experiments is cut from commercial RO membrane unit. Square rubber O-rings are used for sealing. The bottom part contains the permeate channel and two permeate outlet ports. The permeate carrier is placed inside the permeate channel which collects and transports permeate water to permeate ports.

The RO desalination cell is fixed on two shafts and supported by four linear bearings in Fig. 2. A linear actuator (LinMot Inc., P01- $48 \times 360F/60 \times 210$ ) is controlled by a laptop-based program and is used to vibrate the cell at desired amplitude and frequency. For all vibration cases, vibration amplitude is fixed at 1.2 mm while vibration frequency varies from 20 Hz to 50 Hz.

Fig. 3 shows a hydraulic circuit diagram of the test installation. The stainless steel feed tank has a volume of 22.7 L and serves as a supply for the stainless steel diaphragm pump (Hydra-Cell, M03BABTHFECA) which pressurizes the feed solution. The pulsation dampener (CAT, 6028) is used to absorb the feed flow pressure fluctuations. The inlet pressure and flow rate are measured by a digital pressure transducer (Omega, PX309-1KGV) and in-line flow meter just upstream of the desalination cell. The pressure relief valve and regulating valve are used to adjust the cell pressure and feed flow rate, respectively. The retentate flow goes through the regulating valve to reduce pressure to atmosphere and goes back to the feed tank. The permeate flow is collected and measured at the desalination cell side. Permeate flow is poured back to the feed tank after measuring in order to keep the feed water salinity constant. The concentration of the feed water and permeate water was measured by a conductivity meter (Hana Instruments Inc., HI98192), which has an EC accuracy of 0.01 S/cm.

## 2.3. Experiment procedure

A new membrane was used for each experiment and was first compacted using distilled water at an operating pressure of 55 bar for at least 4 h until a stable baseline permeate flux was obtained.

Non-vibration membrane cases with three different inlet flow rate were performed in order to establish the benchmark data to compare with the vibration test cases. Vibration values, permeate flux, permeate salinity, feed pressure and feed water temperature were recorded during each vibration membrane test, while keeping the feed flow rate and feed pressure constant.

A feed pressure of 55 bar was employed for all tests and three different feed flow rates were utilized: 0.3785 lpm, 0.5678 lpm, and 0.7570 lpm. These feed flow rates produced three different inlet Reynolds numbers: Re = 344, 516 and 688, respectively. The feed solution temperature was kept in the temperature range of  $25 \pm 1$  °C and measured by a J-type thermocouple with 0.1 °C accuracy.

#### 3. Theory and CFD numerical simulations

#### 3.1. Governing equations

The hydrodynamics of the transient two-dimensional flow inside a

(1)





Fig. 2. Solid model of test cell arrangement.



spacer-filled RO membrane channel is described by the following governing equations.

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0$$

Momentum equations:



Fig. 3. Experimental system diagram.

 Table 3

 Concentration dependent viscosity, diffusivity and osmotic pressure for single salt solutions.

Solute c (mol $L^{-1}$ )	Density $\rho$ (kg m <sup>3</sup> )	Viscosity $\mu$ ( $\times 10^3$ Pa s)	Diffusivity D ( $\times 10^9 \text{ m}^2 \text{ s}^{-1}$ )	Osmotic pressure $\pi$ (bar)
NaCl	995.7 + 38.54c	1.004 + 0.08c	1.45  for  0.1 < c < 1.5	47.93c
CaCl <sub>2</sub>	995.7 + 89.72c	1.004 + 0.2981c	$(1.176 + 107.5c)/(1 + 95.24c + 26.03c^2)$ for c < 0.132;	$62.35c + 13.14c^2 + 8.993c^3$
	for $c < 0.6$	for $c < 0.6$	$1.134-0.3213c + 0.2319c2-0.1553c^3$ for $0.132 < c < 0.6$	for $c < 0.6$
$Na_2SO_4$	995.7 + 11.91c	1.004 + 0.4094c	$(1.08 + 129.3c)/(1 + 124.5c + 40.92c^2)$ for $c < 0.108$ ;	$56.29c - 28.37c^2 + 23.05c^3$
	for $c < 0.6$	for $c < 0.6$	$1.042-0.3225c + 0.1048c2-0.07547c^3$ for $0.108 < c < 0.6$	for $c < 0.6$
MgSO <sub>4</sub>	995.7 + 11.88c	1.004 + 0.06092c	$0.7288-0.1929c + 0.1517c^2-0.9297c^3$ for $0.05 < c < 0.5$	$33.08c - 178.9c^2 + 942.7c^3$
	for $0.05 < c < 0.5$	for $0.05 < c < 0.5$		for $0.05 < c < 0.5$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = -\frac{\partial P}{\partial x} + \nabla \cdot (\mu \nabla u)$$
(2)

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho u v)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = -\frac{\partial P}{\partial y} + \nabla \cdot (\mu \nabla v)$$
(3)

Solute transport of a non-reacting solution in the RO membrane channel is governed by the convection-diffusion equation:

$$\frac{\partial(\rho c_i)}{\partial t} + \frac{\partial}{\partial x} \left(\rho u c_i - D_i \frac{\partial c_i}{\partial x}\right) + \frac{\partial}{\partial y} \left(\rho v c_i - D_i \frac{\partial c_i}{\partial y}\right) = 0$$
(4)

where,  $c_i$  and  $D_i$  are the concentration and diffusivity of solute *i*.

Please note that, although the flow velocity is below the Mach number, fluid density is not a constant. It depends on the local salt concentrations which is mentioned in Table 3. The density change is relatively small (< 3%) and it is not sufficient to generate pressure waves. Pressure induced density change is neglected.

#### 3.2. Geometry and mesh generation

A rectangular 2D membrane channel filled with 'zigzag' cylindrical spacers was adopted as the computational domain. The simulation domain is 200 mm in length and 0.78 mm in height, which has the same dimensions as the experimental setup described in Section 2.2. The left and right boundaries are the inlet and outlet for this channel. The top wall is impermeable wall while the bottom is a semi-permeable RO membrane.

Each set of two successive spacers comprises one 'cell element' as

shown in Fig. 4. This pattern is repeated 35 times in the simulation domain. The entire computational domain consists of the 35 repeated 'cell elements' and a 0.5 mm length exit area. Each 'cell element', has  $l_m = 2.85$  mm and  $r_s = \frac{1}{4}h = 0.195$  mm. This 'zigzag' geometry is closely similar with the spacer-filled channel in the experiments.

Structured mesh generated using ICEM-CFD (Ansys, Inc., Canonsburg, PA) was adopted in this simulation to obtain accurate results with low computing time. Fig. 5 depicts representative details of the mesh structure. The mesh near the membrane is much finer due to the expected large concentration gradient near its surface.

Four simulations were conducted using  $4.06 \times 10^5$ ,  $6.80 \times 10^5$ ,  $8.48 \times 10^5$  and  $10.6 \times 10^5$  cells in order to eliminate the effect of mesh quality and size on the results. Fig. 6 shows the steady membrane permeate flux profile in the middle of the 18th cell element which is far enough inward to avoid the influence of the inlet and outlet. A mesh with  $6.80 \times 10^5$  cells has a good enough accuracy when predicting membrane permeate flux. Hence, the computational grid was constructed using  $6.80 \times 10^5$  cells. For this mesh size, the first cell height on the membrane boundary and spacer wall boundary are  $0.8 \,\mu\text{m}$  and  $3 \,\mu\text{m}$ , respectively.

#### 3.3. Boundary conditions

The membrane local permeate flux is defined by Darcy's law for porous media:

$$J = K \left( \Delta P - \Sigma \Delta \pi_i \right) \tag{5}$$

where, K is the membrane hydrodynamic permeability,  $\Delta P$  is the



Fig. 4. Model geometry. (A) The whole simulation domain. The dashed box shows one unit cell element. The unit cell element is repeated 35 times in the simulation domain. (B) One unit cell element.



Fig. 5. Representative mesh structure.

applied trans-membrane pressure which equals the difference between the feed channel pressure and permeate pressure and  $\Delta \pi_i$  is the osmotic pressure difference across the membrane for solute *i*.

In this case, K is assumed to be a constant for a given membrane system and can be calculated from pure water experiments. There is no solute inside the system in the pure water experiment. So for this case, the above equation becomes:

$$K = \frac{J}{\Delta P} \tag{6}$$

The constant *K* is obtained using Eq. (6) and measuring *J* and  $\Delta P$  with a flow meter and pressure gauge, respectively.

The following fully developed velocity profile and solutes concentrations are imposed at the inlet (x = 0) as:

$$u = 6\overline{u}\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$v = 0$$

$$c_i = c_{i,b}$$
(7)

where,  $c_{i, b}$  is the inlet bulk concentration of solute *i* and  $\bar{u}$  is the averaged feed inlet velocity which is set to match the corresponding experimental flow condition. The experiment's feed composition shown in Table 1 was also utilized in the simulation.

Three given feed channel Reynolds numbers 344, 516 and 688, corresponding with the experimental inlet flow rates 0.3785 lpm, 0.5678 lpm, and 0.7570 lpm, are considered in the simulations.

The channel Reynolds number is defined as [18]:

$$\operatorname{Re}_{ch} = \frac{\rho \overline{u} d_h}{\mu} \tag{8}$$

where,  $d_h$  is the hydraulic diameter for the spacer-filled membrane channel:



where, *h* is the channel height = 0.78 mm,  $r_f$  is the spacer radius =  $\frac{1}{4}h$  = 0.195 mm,  $\epsilon$  is the voidage of the spacer-filled membrane channel:

$$\varepsilon = \frac{V_{tot} - V_{sp}}{V_{tot}}$$
(10)

and  $V_{tot}$  and  $V_{sp}$  are the total channel volume (channel length × channel height for 2D simulations) and volume occupied by spacers (area of circular spacers cutplane for 2D simulations), respectively.

The pressure outlet boundary condition in the simulations is set equal to the measured outlet pressure of the membrane channel, and the velocity gradient and solute concentration gradient are assumed to be zero since the outlet boundary is far away from the membrane zone.

$$\frac{\partial v}{\partial x} = 0, \quad \frac{\partial c_i}{\partial x} = 0$$
 (11)

The given vibration velocity and no solute flux boundary condition are imposed at the non-membrane wall:

$$u = 2\pi f A \sin(2\pi f t)$$

$$v = 0$$

$$\frac{\partial c_i}{\partial \vec{n}} = 0$$
(12)

where, *A* and *f* are the vibration amplitude and frequency, and *A* is 1.2 mm, and *f* is 20 Hz, 30 Hz, 40 Hz and 50 Hz in the simulations.  $\vec{n}$  is the unit normal vector at the wall boundary. In the FLUENT simulation, the velocity value calculated in Eq. (12) was added as the boundary velocity using UDF code to achieve the boundary vibration in the given manner.

The boundary conditions for x flow velocity equal to vibration velocity, diffusive and convective fluxes being equal, and salt solute

Fig. 6. Membrane permeate flux profile using different mesh sizes.



concentration are imposed at the membrane wall as:

$$u = 2\pi f A \sin(2\pi f t)$$

$$v = -J$$

$$D_i \frac{\partial c_i}{\partial y} = -J c_i R_i$$
(13)

where, *J* is the membrane local permeate flux determined via Eq. (5),  $R_i$  is the intrinsic rejection coefficient of the membrane:

$$R_i = 1 - \frac{c_{i,p}}{c_{i,m}} \tag{14}$$

where,  $c_{i, p}$  and  $c_{i, m}$  are the permeate concentration and membrane local concentration of solute i. In this simulation, the  $c_{i, p}$  are experimentally measured, and the  $c_{i, m}$  are unknown and obtained via the CFD simulation.

#### 3.4. Numerical simulation

Transient simulations were performed using the LES turbulence model in order to obtain an accurate prediction of the vibratory membrane performance in reducing concentration polarization. The LES model separates the turbulent flow field into a combination of the large, energy-containing part and sub-grid part using the Smagorinsky-Lilly subgrid-scale model [19]. The resolved part represents the large eddies which control the flow behavior and the turbulent transport. The subgrid part represents the small scales which are assumed to be isotropic [20].

The space filter which was used to separate large and small scales in the whole domain D has the filtering kernel G [21]:

$$\overline{u}_{i}(\vec{x},t) = \int_{D} G\left(\vec{x} - \vec{\xi}, t\right) u\left(\vec{\xi}, t\right) d(\vec{\xi})$$
(15)

After applying the filtering operation, the filtered Navier-Stokes equations of motion become [22]:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho}\,\bar{u}_j)}{\partial x_j} = 0 \tag{16}$$

$$\frac{\partial(\bar{\rho}\,\tilde{u}_i)}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho}\,\tilde{u}_i\tilde{u}_j + \bar{p}\,\delta_{ij} - \tilde{\sigma}_{ji}) = -\frac{\partial\tau_{ji}}{\partial x_j} \tag{17}$$

where, sub-grid stresses (SGS)  $\tau_{ii}$  are defined as:

$$\tau_{ji} = \overline{\rho} \left( u_j u_i - \widetilde{u}_j \widetilde{u}_i \right) \tag{18}$$

Fluent (Ansys, Inc., Canonsburg, PA) was used here as the CFD based solver of the coupled continuity, momentum and solute transport equations via the SIMPLEC algorithm. A second order upwind scheme is applied to interpolate solute variables, and the momentum equations are spatially discretized with the bounded central differencing approach. Time is discretized using a 2nd order implicit method.

The concentration dependent model parameters such as viscosity, diffusivity, and osmotic pressure are shown in Table 3 [5,23–25]. In this table, c denotes solute concentration in mol  $L^{-1}$  unit.

The Refutas index method [26] was used to predict the blend viscosities of the mixture. In this method, the Refutas index of each component (*VBI<sub>i</sub>*) and the Refutas index of the blend (*VBI<sub>β</sub>*) are calculated using Eqs. (19) and (20). In Eq. (20),  $\omega_i$  is the mass fraction of component *i*.

$$VBI_i = 23.097 + 33.469 \log \log(v_i + 0.8)$$
<sup>(19)</sup>

$$VBI_{\beta} = \sum_{1}^{n} \omega_i VBI_i \tag{20}$$

Very small time steps are required to adequately resolve the unsteady flow inside the vibratory channels for turbulence simulation. A time step of  $2.0 \times 10^{-6}$  s was used for all non-vibration and vibration cases. This value is smaller than time steps used in previous simulations [4,27–29], and the chosen time step also yields a maximum Courant number of approximately 0.2 which is sufficiently small to perform LES simulation.

Simulations were judged to be converged once the absolute mass, momentum, and solute concentration residuals were smaller than  $5 \times 10^{-6}$ ,  $5 \times 10^{-6}$  and  $1 \times 10^{-9}$ , respectively. Higher convergence requirements are needed for solutes concentration since it plays a pivotal role in calculating membrane concentration polarization and permeate flux variation along the membrane surface.

#### 3.5. Post-processing

#### 3.5.1. Concentration polarization modulus

The concentration polarization (CP) modulus of solute *i* is evaluated from the simulated solute concentration  $c_{i, m}$  at the membrane surface as:

$$\Gamma_i(x) = \frac{c_{i,m}}{c_{i,b}} \tag{21}$$

#### 3.5.2. Membrane surface shear rate

The membrane surface x direction shear rate is calculated from:

$$\gamma(x) = \frac{\partial u}{\partial y} \bigg|_{y=0}$$
(22)

#### 3.5.3. Membrane surface Sherwood number

The membrane surface Sherwood number for solute *i* is determined from:

$$Sh_i(x) = \frac{k_m(x)d_h}{D_i}$$
(23)

where,  $k_m(x)$  is the membrane local mass transfer coefficient at position x and is calculated as

$$k_m(x) = \frac{D_i \frac{\partial c_i}{\partial y}\Big|_{y=0}}{c_{i,b} - c_{i,m}}$$
(24)

In Eq. (24),  $c_{i, b}$  and  $c_{i, m}$  are the bulk flow concentration and adjacent membrane concentration of solute *i*, respectively. The parameter  $d_h$  is the hydraulic diameter for a spacer-filled membrane channel which is defined in Eq. (9).

## 4. Results and discussion

Table 4 shows the model parameters used in CFD simulations, including twelve different vibration cases and three non-vibration cases.

The CFD computations were performed for one vibration period in all vibration cases since the vibration period T is much larger than the vortex shedding time and the vibration has a higher influence on the membrane desalination. Non-vibration cases were also studied for 0.02 s as the control group.

#### 4.1. Membrane local concentration polarization profile

*NaCl* is the dominant solute in the feed solution responsible for most part of the salt osmotic pressure. A lower membrane *NaCl* CP modulus means that a lower operation pressure is needed to get the same permeate production for a given membrane system and feed solution. This will reduce the energy cost significantly in the real RO membrane seawater desalination system. Additionally, *CaSO*<sub>4</sub> is a barely soluble mineral salt in the feed solution, which leads to mineral salt scaling on the membrane surface. Therefore, both *NaCl* and *CaSO*<sub>4</sub> membrane concentration polarization profiles were predicted and are shown in this section. Since the solubility of *CaSO*<sub>4</sub> in water is lower than other

Table 4 Summary of CFD simulation conditions.

Run No.	Re	Vibration frequency (Hz)	Vibration amplitude (mm)	Pressure (bar)	Vibration period (s)
1	344	0	NA	55	NA
2	344	20	1.2	55	0.050
3	344	30	1.2	55	0.033
4	344	40	1.2	55	0.025
5	344	50	1.2	55	0.020
6	516	0	NA	55	NA
7	516	20	1.2	55	0.050
8	516	30	1.2	55	0.033
9	516	40	1.2	55	0.025
10	516	50	1.2	55	0.020
11	688	0	NA	55	NA
12	688	20	1.2	55	0.050
13	688	30	1.2	55	0.033
14	688	40	1.2	55	0.025
15	688	50	1.2	55	0.020

possible calcium compounds such as  $CaCl_2$ , gypsum crystals are more eager to appear at the membrane surface compared with  $CaCl_2$  crystals for the given solution and desalination operation used in experiments. The authors use the concentration value of  $Ca^{2+}$  as the amount of potential  $CaSO_4$  dissolved in the solution and use this value to check the possibility of the gypsum formation.

Fig. 7 depicts membrane local *NaCl* concentration polarization modulus variation at the membrane surface with time for fifteen different flow and vibration cases. For three non-vibration cases (black lines), the higher the Reynolds number was, the lower *NaCl* CP modulus was. Increased inlet Reynolds number mitigates the salt accumulation near the membrane for the non-vibration cases. More salt will be transported back into the bulk flow thus avoiding the membrane surface accumulation at the high Reynolds number condition.

Similar with the non-vibration cases, increasing the inlet Re numbers will also reduce the *NaCl* CP modulus for vibration cases while keeping the vibration frequency the same (the same line color). Consideration of the effects of vibration, all vibration cases have lower *NaCl* CP modulus than the corresponding non-vibration cases for all three Re numbers.

From Eqs. (12) and (13), the membrane wall moves with acceleration:

$$a = (2\pi f)^2 A \cos(2\pi f t) \tag{25}$$



As shown in Eq. (25), the vibration acceleration at the membrane boundary forms a cosine curve for all vibration frequencies. The vibration acceleration equals to zero when T = 0.25 and 0.75. The local NaCl CP modulus reaches local maximum at these times. Similarly, vibration acceleration reaches its peak value when T = 0, 0.5 and 1, and the local *NaCl* CP modulus reaches a local minimum at these times. The wave form of *NaCl* CP curves for all vibration cases are similar with sine wave, especially for low Re case. These results strongly indicates that the *NaCl* CP is highly dominated by membrane vibration, especially for low Re case. For the high Re cases, due to the complexity of the fluid flow itself, *NaCl* CP is not strongly affected by membrane vibration compared with low Re cases.

Cases with the higher vibration frequencies have lower *NaCl* CP modulus when keeping the Reynolds number the same (the same line symbol). Increasing the vibration frequency, while keeping the vibration amplitude constant, causes an increase in the overall shear rate at the membrane surface.

Fig. 8 shows the membrane surface  $CaSO_4$  concentration polarization modulus variation with time for different flow and vibration conditions. Similar with the *NaCl* CP profile, higher inlet Reynolds number and higher vibration frequency will result in lower  $CaSO_4$  CP modulus.

Fig. 9 shows the time averaged *NaCl* and *CaSO*<sub>4</sub> concentration polarization modulus at the membrane surface for cases with different vibration frequencies. Similar to the conclusions based on Fig. 7, under the same vibration conditions, cases with higher Reynolds numbers have lower membrane surface salt CP modulus, and with the same inlet Reynolds number, a higher frequency induces a lower salt CP modulus. By comparing *NaCl* and *CaSO*<sub>4</sub> values, it is found that under the same flow and vibration conditions, the *CaSO*<sub>4</sub> CP modulus is slightly higher than *NaCl* CP modulus. This shows that the CaSO<sub>4</sub> accumulation is more severe than *NaCl*. Thus *CaSO*<sub>4</sub> is more difficult for the bulk flow to carry out from the membrane surface area.

Fig. 10 shows the  $CaSO_4$  CP modulus distribution in the middle filament at different times for the Re = 344, 50 Hz vibration case (top) and Re = 344, non-vibration case (bottom). The middle (the 18th) cell element was chosen as the example cell element to visualize detailed flow characteristic, salt solute concentration, and membrane local permeate flux since it is far enough to avoid the influence of the inlet and outlet conditions. From these figures, there is no obvious change of the  $CaSO_4$  CP modulus distribution inside the non-vibration case (bottom figures) and the corresponding maximum  $CaSO_4$  CP modulus is 3.8. For the Re = 344, 50 Hz vibration case (top figures), obvious transient patterns were observed. Salt accumulated on the membrane

Fig. 7. Membrane *NaCl* concentration polarization modulus variation with time (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.).

1 55

Fig. 8. Membrane CaSO<sub>4</sub> concentration polarization modulus variation with time.



Fig. 9. Time averaged membrane NaCl and CaSO<sub>4</sub> concentration polarization modulus.

surface near the spacer was transported back to the bulk flow. The maximum  $CaSO_4$  CP modulus is 2.5 which is much lower than 3.8 in non-vibration case.

Previous investigations [30–32] show that reducing the local  $CaSO_4$  CP modulus will reduce the rate of gypsum precipitation and membrane fouling since the new nuclei nucleation rate and crystal growth rate are related to local  $CaSO_4$  concentration and degree of saturation. Our simulation shows that membrane vibration reduces  $CaSO_4$  concentration at the membrane surface, especially at the membrane-spacer connection area. This  $CaSO_4$  concentration reduction will also reduce the gypsum precipitation and membrane fouling potential.

#### 4.2. Shear rate and detailed flow behavior

Fig. 11 shows the membrane surface shear rate variation with time when Re = 344. As expected there is almost no variation in shear rate for the non-vibration case. The figure also shows that the boundary-peak shear rate significantly increases with vibration frequency.

The root mean square (RMS) value of membrane shear rate was adopted in order to show the effect of vibration on membrane shear rate. The expression for RMS value of the membrane shear rate in one vibration period is shown in Eq. (26). Fig. 12 shows the RMS of membrane boundary shear rate value vs. vibration frequency. It is clear that boundary shear rate is greatly increased by vibration especially for the Re = 344 case. The shear rate increase induced by membrane vibration for Re = 688 is significant but relatively smaller than in the low Re case. This is due to the complexity of the flow condition near the feed spacers with Re = 688.

$$\gamma_{rms} = \sqrt{\frac{1}{T} \int_0^T |\gamma(t)|^2 dt}$$
(26)

Velocity vectors near the downstream face and the upstream face of the membrane spacer are visualized in this section. The physical significance of this is that the fluid behavior in the feed channel especially near the spacers plays a very important role in the salt accumulation and diffusion near the membrane. The Re = 344 and 50 Hz vibration were chosen for the example to show the difference between non-vibration and vibration cases.

Fig. 13 shows the velocity vector profile in the middle filament over 20 ms for the Re = 344, 50 Hz vibration case (top) and Re = 344, non-vibration case (bottom). Similar with previous investigations [4,33], it is useful to label vortices near the bottom spacer area to track their generation, growth, movement and decay over time.

A small recirculation region was formed near the spacer upstream face while a larger recirculation region was located at the spacer downstream area in both non-vibration and vibration cases. Vortices formed in both regions and moved downstream with the bulk flow, and vortex shedding is found in the spacer downstream area. In all subfigures of Fig. 13, two or three successive big vortices can always be found in the downstream area of the spacer, whether the membrane vibrate or not. They tend to move with the bulk flow and slightly towards the membrane. These successive vortices will be called 'dominant downstream' vortices for simplicity in the following discussion.

The membrane boundary velocity is positive during the first half cycle of shaking, t = 0 ms-10 ms, for the 50 Hz vibration case. Near the start of the calculation, when t = 0.8 ms (Fig. 13(a)), three 'dominant downstream' vortices are marked as vortex 1–3. There are no vortices in the upstream side of the spacer, which describes the non-vibration case and the vibration case.

When t = 3 ms (Fig. 13(b)), it is interesting to note that there are two vortices 4 and 5 formed between the membrane and the two 'dominant downstream' vortices for the vibration case. This is mainly due the interaction between positive x-velocity at the membrane surface and negative x-velocity in the bottom of the 'dominant downstream' vortices. Two parallel vortices 6 and 7 are created at the upstream side of the membrane spacer, and have the opposite rotation



(c) t = 13.8 ms

Fig. 10. CaSO<sub>4</sub> CP modulus distribution in the middle filament when (a) t = 5 ms, (b) t = 10 ms and (c) t = 13.8 ms for 50 Hz vibration (top) and non-vibration case (bottom).



Fig. 11. Membrane boundary shear rate variation with time for Re = 344 cases.



Fig. 12. RMS of membrane boundary shear rate vs. vibration frequency.



(e) t = 10 ms

Fig. 13. Velocity vector profile in the middle filament during one vibration time period (20 ms) for 50 Hz vibration (top) and non-vibration case (bottom). Shed vortices are identified by numbers 1–15.





direction. These two vortices will enhance the membrane local salt mass transfer and reduce the concentration polarization. Low membrane local velocity will reduce the salt mass transportation process. There are no vortices 4 and 5 formed near the membrane for non-vibration case. Another difference is that there is only one vortex 6 formed at the upstream face for the non-vibration case compared with two vortices in the vibration case. In the vibration case, the different velocity direction between the bottom of vortex 6 and membrane boundary induces vortex 7, increases flow complexity and enhances local mass transport. Similar with Fig. 13(b), in Fig. 13(c) and Fig. 13(d), more vortices appear near the membrane in the vibration case compared with the corresponding non-vibration case. The membrane boundary velocity equals to 0 when t = 10 ms (Fig. 13(e)), which is the same as the non-vibration case.

From Fig. 13(e) to Fig. 13(i), the difference in the number of vortices between the vibration case and the non-vibration case is not that

Fig. 14. Membrane NaCl Sherwood number variation with time.



obvious as in the time period: 0 ms-10 ms. The reason is that during t = 10 ms-20 ms, the membrane boundary velocity will become negative which is the same direction as the x-velocity of the fluid in the bottom of the 'dominant downstream' vortices.

Summarizing key observations: (a) flow in both the vibration and non-vibration cases follow the 'zigzag' pattern formed by the spacer geometry, (b) significant unsteady flow behavior occurs for the nonvibration and vibration cases, (c) imposed boundary vibration spawns vortex generation and decay at the membrane surface and also increases the boundary shear rate, and (d) as a result of imposed boundary vibration salt mass transfer is enhanced, salt CP modulus is reduced and membrane permeate flux is increased.

#### 4.3. Membrane local mass transfer and Sherwood number

The Sherwood number is widely used when evaluating membrane performance since it eliminates the effect of inlet conditions and channel length. By calculating the Sherwood number, we can evaluate the salt mass transfer especially near the membrane surface for different inlet Reynolds numbers and different vibration frequencies.

Fig. 14 depicts membrane surface *NaCl* Sherwood number profile variation with time for different flow and vibration conditions. The *NaCl* Sherwood number is seen to increase with Reynolds number by considering the three non-vibration cases (black lines). Increased inlet Reynolds number will increase the membrane local salt mass transport rate to the bulk flow. All vibration cases show better Sherwood number profile compared with corresponding non-vibration case. A second trend is that the higher vibration frequency, the higher the *NaCl* Sherwood number.

Fig. 15 shows membrane surface  $CaSO_4$  Sherwood number profile variation with time for different flow and vibration conditions. Fig. 16 shows the time averaged membrane *NaCl* and *CaSO*<sub>4</sub> Sherwood number profile vs. vibration frequency.

Time averaged *NaCl* Sherwood number is increased by vibration for all Re numbers. The maximum Sherwood number increment is around 13% for the 50 Hz vibration cases. *CaSO<sub>4</sub>* Sherwood number profile shows similar trends with *NaCl* Sherwood number profile. By comparing time averaged *NaCl* and *CaSO<sub>4</sub>* Sherwood number profile, it is found that under the same flow and vibration conditions, *CaSO<sub>4</sub>* Sherwood number is slightly higher than *NaCl* Sherwood number. For the concentration appearing in this simulation set, the diffusivity of *CaSO<sub>4</sub>* is much lower than *NaCl* as shown in Table 3. Since the Sherwood number is the ratio of convective mass transfer rate and diffusive mass transport rate shown in Eq. (23), even with a higher Sherwood number, the  $CaSO_4$  solution convective mass transfer rate  $k_m(CaSO_4)$  is still lower than NaCl which means  $CaSO_4$  is easier to accumulate at the membrane surface. This also explains the phenomenon found in the concentration polarization section, namely that the CP modulus of  $CaSO_4$  is higher than NaCl.

#### 4.4. Membrane local permeate flux profile

Fig. 17 shows normalized permeate flux variation with time for different flow and vibration conditions. Permeate fluxes for all cases were compared with the Re = 344, no vibration case to get the normalized permeate flux for better understanding of the effect of both Re numbers and vibrations. Eq. (5) and Table 3 show that the higher the membrane local salt concentration, the lower the membrane permeate flux.

Consistent with the CP modulus profile shown in Fig. 9, the time averaged permeate flux for vibration cases are higher than for the non-vibration cases over all Re numbers.

## 4.5. Comparison between CFD simulation results and experimental results

Fig. 18 shows very good agreement between simulation and experiment derived time averaged normalized permeate flux. The zero frequency point in this figure corresponds to the no vibration case. The trend shows that the higher the vibration frequency, the higher the permeate flux. By comparing the 50 Hz and non-vibration cases for Re = 516, a maximum increase of 10% was found. As expected, Fig. 17 and Fig. 18 show similar trends for the dependency of permeate flux on Re and frequency.

## 5. Conclusions

The effectiveness of a new approach to desalination utilizing RO membrane vibration was confirmed utilizing experimentation with artificial seawater and by CFD simulations including coupled motion and mass transfer equations. The CFD simulations provided highly accurate, time-dependent membrane surface permeate flux distribution, *NaCl* and *CaSO*<sub>4</sub> concentration polarization profiles, and mass transfer and flow patterns in the spacer-filled channel over one vibration time period. The CFD derived transient flow patterns were intuitively consistent for the non-vibration and vibration cases. The CFD predicted eddies near the membrane surface in the vibration cases reduced the *NaCl* and *CaSO*<sub>4</sub>

110

- Re = 344, Simulation

Re = 516, Simulation

- Re = 516, Experiment - Re = 688, Experiment

Re = 688, Simulation
Re = 344, Experiment

Fig. 15. Membrane CaSO<sub>4</sub> Sherwood number variation with time.



Fig. 16. Time averaged membrane NaCl and CaSO4 Sherwood number vs. vibration frequency.



Fig. 17. Normalized permeate flux variation with time.

40

30

50

concentration polarization and consequently increased the membrane mass transfer rate. The reduced membrane  $CaSO_4$  concentration polarization resulting from the imposed vibration also reduces the membrane local gypsum fouling rate. The simulations showed that the CP modulus decreases and the permeate flux increases as the vibration frequency increases, with a constant amplitude imposed vibration. The improvements in CP and flux change rapidly up to 20 Hz. and then slow considerably for higher frequencies. This suggests that in tuning the vibratory system it is important to locate and use the minimum frequency above which added benefits are relatively small. This will save power and equipment costs. This result was also confirmed by experimental measurements.

## Nomenclature

- x axial coordinate (cross-flow direction) (m) (m s<sup>-1</sup>)
- *y* lateral coordinate (channel height direction) (m)
- *u* velocity component in the x-direction  $(m s^{-1})$
- v velocity component in the y-direction (m s<sup>-1</sup>)
- *D* diffusion coefficient  $(m^2 s^{-1})$
- c solute concentration (mol  $L^{-1}$ )
- $\Delta P$  transmembrane pressure (Pa)
- *K* hydrodynamic permeability  $(m^2 s kg^{-1})$
- J permeate flux (m s<sup>-1</sup>)
- *h* feed channel height (m)
- $\overline{u}$  averaged x-direction velocity (m s<sup>-1</sup>)
- Re Reynolds number
- *f* vibration frequency (Hz)
- *A* vibration amplitude (mm)
- Sh Sherwood number
- k mass transfer coefficient (m s<sup>-1</sup>)
- *d<sub>h</sub>* hydraulic diameter (mm)
- *r<sub>f</sub>* spacer radius (mm)

#### Greek symbols

- $\rho$  solution density (kg m<sup>-3</sup>)
- $\mu$  dynamic viscosity of the solution (Pa s)
- $\pi$  solute osmotic pressure (Pa)
- Γ concentration polarization modulus
- $\gamma$  shear rate  $(s^{-1})$
- ω

#### Subscripts

- i solute i
- *b* bulk flow
- *m* solution adjacent to the membrane
- *p* permeate flow

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